Preface

> *"Wavelet theory" is the result of a multidisciplinary effort that brought together mathematicians, physicists and engineers...this connection has created a flow of ideas that goes well beyond the construction of new bases or transforms.*
>
> —Stéphane Mallat[1]

The past two decades have witnessed an explosion of activity in wavelet analysis. Thousands of research papers have been published on the theory and applications of wavelets. Wavelets provide a powerful and remarkably flexible set of tools for handling fundamental problems in science and engineering. With the second edition of this primer, I aim to provide the most up-to-date introduction to this exciting field.

What is new in this second edition?

I have added a lot of new material to this second edition. The most important additions are the following:

- *Exercises and additional examples:* At the end of the chapters, I provide a total of 104 worked examples, and 222 exercises (with solutions provided for representative problems). These examples and exercises constitute a book-in-itself of review material, and should make this primer a valuable text for courses on wavelets.
- *Biorthogonal wavelets:* I have added two sections that describe the most important examples of these wavelets, and I describe their application to image compression.

[1]Mallat's quote is from [4] listed in the Notes and references section for Chapter 3.

- *Image compression:* I have included a mini-course on image compression, with descriptions of two state-of-the art algorithms (ASWDR and JPEG 2000). This discussion includes a tutorial on arithmetic compression, an important topic that is omitted from most (if not all) introductions to image compression.

- *Image denoising:* I have included a lot more material on image denoising, including a description of a high-performance wavelet denoiser. In addition to the standard topic of removing Gaussian random noise, which most books limit themselves to, I also describe a technique for removing isolated, randomly positioned, clutter from images.

- *Gabor transforms and time-frequency analysis:* I provide a concise introduction to the fundamentals of time-frequency analysis and show how it is applied to musical theory, musical synthesis, and audio denoising. An important new mathematical concept in musical theory, known as the *multiresolution principle,* is described here for the first time in book form. I also provide an extensive list of references to the field of time-frequency analysis, a topic that is given short shrift in most introductions to wavelets.

- *Project ideas:* I provide a list, with brief descriptions, of a set of research projects on wavelets and time-frequency analysis. See Appendix A.

- *Book website:* To keep the book as current as possible, and to provide a wealth of additional online material such as software, sound and image files, and links to other web resources on wavelets, I have created a website for the book. I describe this website in more detail later in the preface.

- *Enhanced list of references:* I have more than doubled the list of references for the book; it now totals about 150 items. At the website for the book I provide an online version of this bibliography with a great many links for free downloading of important papers on wavelets and time-frequency analysis.

What problems can be solved with wavelets?

For an idea of the wide range of problems that can be solved with wavelets, here is a list of some of the problems that I shall discuss:

- *Audio denoising:* Long distance telephone messages, for instance, often contain significant amounts of noise. How do we remove this noise?

- *Signal compression:* The efficient transmission of large amounts of data, over the Internet for example, requires some kind of compression. How do we compress this data without losing significant information?

- *Object detection:* What methods can we use to pick out a small image, say of an aircraft, from the midst of a larger more complicated image?
- *Image denoising:* Images formed by microscopes and other devices are often contaminated by large amounts of unwanted clutter (referred to as *noise*). Can this noise be removed in order to clarify the image?
- *Image enhancement:* When an optical microscope image is recorded, it often suffers from blurring. How can the appearance of the objects in these images be sharpened?
- *Image recognition:* How do humans recognize faces? Can we teach machines to do it?
- *Diagnosing heart trouble:* Is there a way to detect abnormal heartbeats, hidden within a complicated electrocardiogram?
- *Speech recognition:* What factors distinguish consonants from vowels? How do humans recognize different voices?
- *Musical analysis.* How can mathematics help us better understand the nature of musical composition?

All of these problems can be tackled using wavelets. I will show how during the course of this book.

Why do we need another wavelet book?

The goal of this primer is to guide the reader through the main ideas of wavelet analysis in order to facilitate a knowledgeable reading of the present research literature, especially in the applied fields of audio and image processing and biomedicine. Although there are many excellent books on the theory of wavelets, these books are focused on the construction of wavelets and their mathematical properties. Furthermore, they are all written at a graduate school level of mathematical and/or engineering expertise. There remains a real need for a basic introduction, a *primer,* which uses only elementary algebra and a smidgen of calculus to explain the underlying ideas behind wavelet analysis, and devotes the majority of its pages to explaining how these underlying ideas can be applied to solve significant problems in audio and image processing and in biology and medicine.

To keep the mathematics as elementary as possible, I focus on the discrete theory. It is in the continuous theory of wavelet analysis where the most difficult mathematics lies; yet when this continuous theory is applied it is almost always converted into the discrete approach that I describe in this primer. Focusing on the discrete case will allow us to concentrate on the applications of wavelet analysis while at the same time keeping the mathematics under control. On the rare occasions when we need to use more advanced mathematics, *I shall mark these discussions off from the main text by putting them* *into subsections that are marked by asterisks in their titles.* An effort has been made to ensure that subsequent discussions do not rely on this more advanced material.

Chapter summaries

Chapter 1 summarizes our subject. Using examples from imaging and audio, I briefly explain wavelet analysis in non-mathematical terms.

In Chapter 2 I introduce the simplest wavelets, the Haar wavelets. I also introduce many of the basic concepts—wavelet transforms, energy conservation and compaction, multiresolution analysis, compression and denoising—that will be used in the remainder of the book. For this reason, I devote more pages to the theory of Haar wavelets than perhaps they deserve alone; keep in mind that this material will be amplified and generalized throughout the remainder of the book.

In Chapter 3 I describe the Daubechies wavelets, which have played a key role in the explosion of activity in wavelet analysis. After a brief introduction to their mathematical properties, I describe several applications of these wavelets. First, I explain in detail how they can be used to compress audio signals—this application is vital to the fields of telephony and telecommunications. Second, I describe how the method of *thresholding* provides a powerful technique for removing random noise (static) from audio signals. Removing random noise is a fundamental necessity when dealing with all kinds of data in science and engineering. The threshold method, which is analogous to how our nervous system responds only to inputs above certain thresholds, provides a nearly optimal method for removing random noise. Besides random noise, Daubechies wavelets can also be used to remove isolated "pop-noise" from audio. The chapter concludes with an introduction to biorthogonal wavelets, which have played an important part in wavelet image compression algorithms such as JPEG 2000.

Wavelet analysis can also be applied to images. In Chapter 4 I provide a mini-course on wavelet-based image compression. I describe two state-of-the-art image compression algorithms, including the new JPEG 2000 standard. I also discuss wavelet-based image denoising, including examples motivated by optical microscopy and scanning tunnelling microscopy. Chapter 4 concludes with some examples from image processing. I examine edge detection, and the sharpening of blurred images, and an example from computer vision where wavelet methods can be used to enormously increase the speed of identification of an image.

Chapter 5 relates wavelet analysis to frequency analysis. Frequency analysis, also known as *Fourier analysis,* has long been one of the cornerstones of the mathematics of science and engineering. I shall briefly describe how wavelets are characterized in terms of their effects on the frequency content of signals. One application that I discuss is object identification—locating a small object within a complicated scene—where wavelet analysis in concert with Fourier analysis provides a powerful approach. The chapter concludes with a short introduction to the theory and application of Gabor transforms, an important relative of wavelet transforms. In recent years Gabor transforms have been increasingly used in sound processing and denoising, where they may be even more useful than wavelet transforms.

In the final chapter I deal with some extensions which reach beyond the fundamentals of wavelets. I describe a generalization of wavelet transforms known as *wavelet packet transforms.* I apply these wavelet packet transforms to compression of audio signals, images, and fingerprints. Then I turn to the subject of *continuous wavelet transforms*, as they are implemented in a discrete form on a computer. Continuous wavelet transforms are widely used in seismology and have also been used very effectively for analyzing speech and electrocardiograms. We shall also see how they apply to analyzing musical rhythm.

Unfortunately, for reasons of space, I could not examine several important aspects of wavelet theory, such as local cosine series, best-basis algorithms, or fast matrix multiplication. At the end of the final chapter, in its Notes and references section, I provide a guide to the literature for exploring omitted topics.

Computer software

Without question the best way to learn about applications of wavelets is to experiment with making such applications. This experimentation is typically done on a computer. In order to simplify this computer experimentation, I have created free software, called FAWAV. Further details about FAWAV can be found in Appendix C; suffice it to say for now that it is designed to allow the reader to duplicate all of the applications described in this primer and to experiment with other ideas. In this second edition I have added several new features, including image compression and denoising of both images and audio, which should add to the usefulness of FAWAV for digital signal processing.

Notes and references

In the Notes and references sections that conclude each chapter, I provide the reader with ample references where further information can be found. These references are numbered consecutively within each chapter. At the risk of some duplication, but with the reader's convenience in mind, I have also compiled all references into a comprehensive Bibliography.

Primer website

To keep the material in the book as up-to-date as possible, and to facilitate classroom use, there is a website for this book at the following address:

`http://www.uwec.edu/walkerjs/Primer/`

This website contains a lot of supplementary material for the book, including the following:

- The latest version of FAWAV and its *User Manual.* The *User Manual* is a comprehensive introduction to the use of FAWAV for digital signal processing.
- Papers for downloading, including links to all papers cited in the primer.
- Sound and image files for use with FAWAV, including all the sound and image files referred to in the primer.
- A collection of *Project ideas* for carrying out your own research. This online collection will include an archive of preprints and publications that deal with these projects, and supplement the list from the primer with new topics.
- Links to other websites dealing with wavelet and time-frequency analysis, including all the websites listed in the primer.

Acknowledgments

It is a pleasure to acknowledge several people who have helped me in writing this book. First, I must thank my wife Angela for her constant love and support. I would also like to thank my executive editor, Bob Stern, for his encouragement and help. I appreciate several suggestions from Steve Krantz that helped me put some finishing touches on the book. Thanks to Thomas Maresca for some enlightening discussion on FAWAV and its use in this primer. Thanks also to the UWEC Office of University Research and Creative Activities for a grant that gave me some extra time away from my teaching responsibilities for work on this project. Finally, my students in my Digital Signal Processing and Digital Image Processing courses deserve a warm thank you for all of their many suggestions and questions, which helped me to write a better book. I especially would like to thank Xiaowen Cheng, Jarod Hart, Nicholas Nelson, Derek Olson, and Elizabeth Otteson for their contributions.

James S. Walker
Eau Claire, Wisconsin

Contents